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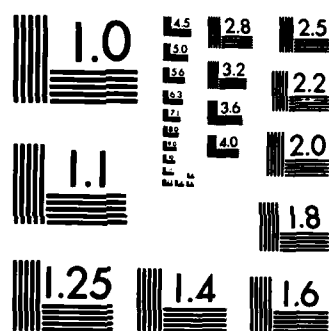
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Norman Draper and Irwin Guttman*

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ABSTRACT

Suppose in a distribution problem, the sample information \underline{W} is split into two pieces \underline{W}_1 and \underline{W}_2 , and the parameters involved are split into two sets, ϕ containing the parameters of interest, and θ containing nuisance parameters. It is shown that, under certain conditions, the posterior distribution of ϕ does not depend on the data \underline{W}_2 , which can thus be ignored. This also has consequences for the predictive distribution of future (or missing) observations. In fact, under similar conditions, the predictive distributions using \underline{W} or just \underline{W}_1 are identical.

AMS (MOS) Subject Classifications: 62F15

Key Words: Bayes' theorem, Nuisance parameters, Predictive distribution

Work Unit Number 4 (Statistics and Probability)

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SIGNIFICANCE AND EXPLANATION

In the application of Bayesian methods, some posterior and predictive distributions may be unaffected when portions of the data are ignored. Conditions under which this is true are given, and examples are provided.

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DROPPING OBSERVATIONS WITHOUT AFFECTING POSTERIOR AND

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Norman Draper and Irwin Guttman*

1. MAIN RESULT

Suppose, that $\underline{W} = (\underline{W}_1', \underline{W}_2')'$ is a vector of random variables whose density function $f_{\underline{W}}(\underline{w}|\underline{\phi}, \underline{\theta})$ depends on two sets of parameters $\underline{\phi}, \underline{\theta}$. Suppose further that we are interested in $\underline{\phi}$, that $\underline{\theta}$ will be regarded as a vector of nuisance parameters and that the following conditions hold.

1. \underline{W}_1 and \underline{W}_2 are statistically independent.
2. The marginal distribution $f_{\underline{W}_2}(\underline{w}_2|\underline{\theta})$ of \underline{W}_2 depends only on $\underline{\theta}$ and not on $\underline{\phi}$.
3. The marginal distribution of $\underline{W}_1 = (\underline{W}_{11}', \underline{W}_{12}')'$ is such that

$$f_{\underline{W}_1}(\underline{w}_1|\underline{\phi}, \underline{\theta}) = f_{\underline{W}_{11}}(\underline{w}_{11}|\underline{\theta}) f_{\underline{W}_{12}|\underline{W}_{11}}(\underline{w}_{12}|\underline{w}_{11}; \underline{\phi}) \quad (1.1)$$

4. The prior information about the parameter sets $\underline{\phi}$ and $\underline{\theta}$ is such that

$$p(\underline{\phi}, \underline{\theta}) = a(\underline{\theta})b(\underline{\phi}) \quad (1.2)$$

so that $\underline{\theta}, \underline{\phi}$ are independent a priori.

Theorem 1. Under conditions 1-4, the marginal posterior of $\underline{\phi}$ based on \underline{W}_1 and \underline{W}_2 does not depend on \underline{W}_2 .

Proof. The posterior of $\underline{\phi}$ given $(\underline{W}_1, \underline{W}_2)' = (\underline{w}_1, \underline{w}_2)'$ is

$$p(\underline{\phi}|\underline{w}_1, \underline{w}_2) \propto \int_{\underline{\theta}} p(\underline{\phi}, \underline{\theta}) f_{\underline{W}_1, \underline{W}_2}(\underline{w}_1, \underline{w}_2|\underline{\phi}, \underline{\theta}) d\underline{\theta} \quad (1.3a)$$

$$\propto b(\underline{\phi}) \int_{\underline{\theta}} a(\underline{\theta}) f_{\underline{W}_2}(\underline{w}_2|\underline{\theta}) f_{\underline{W}_1}(\underline{w}_1|\underline{\phi}, \underline{\theta}) d\underline{\theta} \quad (1.3b)$$

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$$= b(\phi) f_{w_{12}|w_{11}}(w_{12}|w_{11}; \phi) \times$$

$$\int_{\theta} a(\theta) f_{w_2}(w_2|\theta) f_{w_{11}}(w_{11}|\theta) d\theta. \quad (1.3c)$$

The integral in (1.3c) is clearly a function of w_2 and w_{11} , i.e., constant with respect to ϕ , and so may be absorbed into the constant of proportionality.

This proves the result stated.

2. TWO EXAMPLES

Example 2.1. Consider the bivariate normal distribution with vector of means $\underline{\mu} = (\mu_1, \mu_2)'$, variance-covariance matrix $\underline{\Sigma} = ((\sigma_{ij}))$, and inverse $\underline{\Sigma}^{-1} = ((c_{ij}))$.

Let

$$\begin{aligned} \alpha &= \mu_2 + c_{12}\mu_1/c_{22}, & \mu_1 &= \eta_1, \\ \beta &= -c_{21}/c_{22}, & \mu_2 &= \alpha + \beta\eta_1, \\ d_{11} &= c_{22}, & \text{or } c_{11} &= d_{22} + \beta^2 d_{11}, \quad (2.1) \\ d_{22} &= c_{11} - c_{21}^2/c_{22}, & c_{22} &= d_{11}, \\ \eta_1 &= \mu_1, & c_{21} &= -\beta d_{11}. \end{aligned}$$

We remark in passing that, if (2.1) is considered as a transformation from $(\mu_1, \mu_2, c_{11}, c_{22}, c_{21})$ to $(\eta_1, \alpha, \beta, d_{11}, d_{22})$, then the Jacobian has absolute value d_{11} .

Using (2.1), we can re-write the usual form of the bivariate normal frequency function in x_1 and x_2 as

$$f(x_1, x_2 | \mu_1, \alpha, \beta, d_{11}, d_{22})$$

$$= f(x_2 | x_1; \alpha, \beta, d_{11}) \times f(x_1 | \eta_1, d_{22}) \quad (2.2)$$

where

$$f(x_2|x_1; \alpha, \beta, d_{11}) = (d_{11}/2\pi)^{1/2} \exp\{-\frac{1}{2}d_{11}(x_2 - \alpha - \beta x_1)^2\}, \quad (2.2a)$$

$$f(x_1|\eta_1; d_{22}) = (d_{22}/2\pi)^{1/2} \exp\{-\frac{1}{2}d_{22}(x_1 - \mu_1)^2\}. \quad (2.2b)$$

Now suppose that our data on x_1 and x_2 are divided up into two independent portions, satisfying condition 1 of Theorem 1:

\underline{W}_1 , consisting of n independent observations (x_{1i}, x_{2i}) ,
 $i = 1, \dots, n$, on both x_1 and x_2 .

\underline{W}_2 , consisting of n^* independent observations x_{1j}^* ,
 $j = 1, \dots, n^*$, on x_1 alone; the $x_{2j}^* = y_j$ are not observed.

This is the well known missing observations problem (see Draper and Guttman, 1977, and prior references listed therein). In the notation of Section 1, we write

$$\underline{\phi} = (\alpha, \beta, d_{11})' \quad (2.4)$$

for the vector of parameters of interest, and

$$\underline{\theta} = (\eta_1, d_{22})' \quad (2.5)$$

for the vector of nuisance parameters. To obtain a prior for $(\underline{\phi}, \underline{\theta})$, we transform the usual non-informative prior

$$p(\mu_1, \mu_2, c_{11}, c_{22}, c_{12}) \propto (c_{11}c_{22} - c_{12}^2)^{-3/2} \quad (2.6)$$

using (2.1), remembering to insert the Jacobian. This provides

$$p[(\alpha, \beta, d_{11}), (\mu_1, d_{22})] \propto d_{11}^{-1/2} d_{22}^{-3/2} \quad (2.7)$$

from which it is clear that condition 4 of Theorem 1 is satisfied. If we define

$$\underline{W}_{11} = \{x_{1i}, \quad i = 1, 2, \dots, n\}, \quad (2.8)$$

$$\underline{W}_{12} = \{x_{2i}, \quad i = 1, 2, \dots, n\},$$

and employ (2.2) to (2.5), condition 3 of Theorem 1 is satisfied, and condition 2 is satisfied because of (2.2b). It follows that Theorem 1 applies and that the posterior of $\underline{\phi}$ given $(\underline{W}_1, \underline{W}_2)$ depends only on \underline{W}_1 . In fact it can be verified directly that

$$p(\underline{\phi} | \underline{W}_1, \underline{W}_2) \propto d_{11}^{(n-1)/2} \exp[-\frac{1}{2} d_{11} \{S + (\underline{\phi}_1 - \hat{\underline{\phi}}_1)' \underline{X}' \underline{X} (\underline{\phi}_1 - \hat{\underline{\phi}}_1)\}], \quad (2.9)$$

where

$$S = \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i)^2, \quad \underline{\phi}_1 = (\alpha, \beta)', \quad (2.10)$$

$$\underline{X} = (\underline{1}, \underline{x}_1),$$

$\underline{1}$ is an $n \times 1$ vector of ones, $\underline{x}_1 = (x_{11}, x_{12}, \dots, x_{1n})'$, and

$$\hat{\phi}_1 = (\underline{X}'\underline{X})^{-1}\underline{X}'(x_{21}, x_{22}, \dots, x_{2n})'. \quad (2.11)$$

We note that (2.9) is what we would have obtained if we had calculated $p(\phi|\underline{W}_1)$ on the basis of \underline{W}_1 alone, ignoring \underline{W}_2 .

In the missing observations problem, we are usually concerned with inference on the difference $\delta = \mu_1 - \mu_2$. This can involve use of the predictive distribution

$$h(\underline{y}|\underline{W}_1, \underline{W}_2) = \int \cdots \int f(\underline{y}|\phi, \underline{x}_1^*) p(\phi|\underline{W}_1, \underline{W}_2) d\phi,$$

and we see from the above that $p(\phi|\underline{W}_1, \underline{W}_2)$ can be replaced by $p(\phi|\underline{W}_1)$ with the same result. (Details are given by Draper and Guttman, 1977).

This seems intuitively reasonable from (2.2a) and (2.2b). The distribution of \underline{y} conditional on \underline{x}_1^* depends on α, β and d_{11} , and the distribution of \underline{x}_1^* depends only on η_1 and d_{22} and provides no information on $\phi = (\alpha, \beta, d_{11})'$. Thus, in making inferences about ϕ , \underline{x}_2^* can effectively be ignored.

Example 2.2. In this example, we suppose that (x_1, \dots, x_k) has the multinomial distribution

$$m_k(x_1, \dots, x_k) = \frac{n!}{x_1! \cdots x_k! (n - x_1 - \cdots - x_k)!} \tau_1^{x_1} \cdots \tau_k^{x_k} (1 - \tau_1 - \cdots - \tau_k)^{n - x_1 - \cdots - x_k} \quad (2.12)$$

where $0 \leq x_i \leq n$, $0 \leq \sum_{i=1}^k x_i \leq n$, $0 < \tau_i < 1$, and $\sum_{i=1}^k \tau_i = 1$.

Let $(1 \leq k_1 \leq k-1)$

$$\begin{aligned}
 \theta_1 &= \mathcal{T}_1 & \mathcal{T}_1 &= \theta_1 \\
 \theta_2 &= \mathcal{T}_2 & \mathcal{T}_2 &= \theta_2 \\
 &\vdots & &\vdots \\
 \theta_{k_1} &= \mathcal{T}_{k_1} & \mathcal{T}_{k_1} &= \theta_{k_1} \\
 \phi_1 &= \mathcal{T}_{k_1+1} / (1 - \sum_{i=1}^{k_1} \mathcal{T}_i) & \text{OR} & \mathcal{T}_{k_1+1} = \phi_1 (1 - \sum_{i=1}^{k_1} \theta_i) \\
 &\vdots & &\vdots \\
 \phi_{k_2} &= \mathcal{T}_k / (1 - \sum_{i=1}^{k_1} \mathcal{T}_i) & & \mathcal{T}_k = \phi_{k_2} (1 - \sum_{i=1}^{k_1} \theta_i)
 \end{aligned} \tag{2.13}$$

where $k_2 = k - k_1$.

We note that (2.13), viewed as a transformation from $(\mathcal{T}_1, \dots, \mathcal{T}_k)$ to $(\theta_1, \dots, \theta_{k_1}, \phi_1, \dots, \phi_{k_2})$, has Jacobian whose absolute value is $(1 - \sum_{i=1}^{k_1} \theta_i)^{k_2}$.

Now, as is well known, we may write (2.12) as

$$m_k(x_1, \dots, x_k) = m_k(x_1, \dots, x_{k_1}) f(x_{k_1+1}, \dots, x_k | x_1, \dots, x_{k_1}) \tag{2.14}$$

where

$$\begin{aligned}
 m_{k_1}(x_1, \dots, x_{k_1}) &= \frac{n!}{x_1! \dots x_{k_1}! (n - x_1 - \dots - x_{k_1})!} \mathcal{T}_1^{x_1} \dots \mathcal{T}_{k_1}^{x_{k_1}} (1 - \mathcal{T}_1 - \dots - \mathcal{T}_{k_1})^{n - x_1 - \dots - x_{k_1}} \\
 f(x_{k_1+1}, \dots, x_k | x_1, \dots, x_{k_1}) &= \frac{(n - x_1 - \dots - x_{k_1})!}{x_{k_1+1}! \dots x_k! (n - \sum_{i=1}^k x_i)!} \phi_1^{x_{k_1+1}} \dots \phi_{k_2}^{x_{k_1+k_2}} \\
 &\quad \times (1 - \phi_1 - \dots - \phi_{k_2})^{n - \sum_{i=1}^k x_i}
 \end{aligned} \tag{2.14a}$$

It is often the case that, given x_1, \dots, x_{k_1} , the probability of observing characteristic k_1+j , $j=1, \dots, k_2$, is of interest and,

of course, $\phi_j = T_{k_1+j} / (1 - \sum_{i=1}^{k_1} T_i)$ is this conditional probability. Suppose that in gathering data to estimate $\phi = (\phi_1, \dots, \phi_{k_2})'$, that the data on (x_1, \dots, x_k) has two independent pieces -

$$W_1 = (x_1^{(1)}, x_2^{(1)}, \dots, x_k^{(1)})',$$

and

(2.16)

$$W_2 = (x_1^{(2)}, \dots, x_{k_1}^{(2)})',$$

where W_1 has probability function given by (2.14) and W_2 has probability function given by (2.14a), with $1 \leq k_1 \leq k-1$. Suppose too that a-priori,

$$p(T_1, \dots, T_k) \propto c \quad (2.17)$$

Hence, consulting (2.13), and recalling that ϕ is of interest and θ is nuisance, we find

$$p(\phi, \theta) \propto (1 - \sum_{i=1}^{k_1} \theta_i)^{k_2} \quad (2.17a)$$

Hence, condition 4 of Theorem 1 is satisfied, and letting

$$W_1 = \begin{pmatrix} x_1^{(1)} & \dots & x_{k_1}^{(1)} \\ \vdots & & \vdots \\ x_{k_1+1}^{(1)} & \dots & x_k^{(1)} \end{pmatrix} = (W_{11}' : W_{12}'), \text{ it is easy to}$$

see that conditions 1-3 of Theorem 1 also hold. Hence, from Theorem 1, the marginal posterior of ϕ , given W_1, W_2 , does not depend on W_2 . Indeed, it is easy to see directly that the posterior of ϕ , given W_1 and W_2 is

$$p(\phi|\tilde{w}_1, \tilde{w}_2) \propto \phi_1^{x_{k_1+1}^{(1)}} \dots \phi_{k_2}^{x_{k_1+k_2}^{(1)}} (1-\phi_1-\dots-\phi_{k_2})^{n-\sum_{i=1}^k x_i^{(1)}}$$

so that $p(\phi|\tilde{w}_1, \tilde{w}_2) = p(\phi|\tilde{w}_1) = p(\phi|\tilde{w}_{11}, \tilde{w}_{12})$.

3. A SUBSIDIARY RESULT.

In this section, we suppose that the data $\underline{W} = (\underline{W}_1, \underline{W}_2)$, with \underline{W}_1 independent of \underline{W}_2 , is such that

1. The density function of \underline{W}_1 is such that

$$f_{\underline{W}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2, \underline{\theta}) = f_{\underline{W}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2) \quad (3.1)$$

and the density function of \underline{W}_2 , is such that

$$f_{\underline{W}_2}(\underline{w}_2 | \underline{\phi}_1, \underline{\phi}_2, \underline{\theta}) = f_{\underline{W}_2}(\underline{w}_2 | \underline{\theta}) \quad (3.2)$$

2. $p(\underline{\phi}_1, \underline{\phi}_2, \underline{\theta}) \propto a(\underline{\theta}) b(\underline{\phi}_1, \underline{\phi}_2)$

We suppose interest is in $\underline{\phi}_1$ alone, so that $\underline{\theta}$ and $\underline{\phi}_2$ are vectors of nuisance parameters. We have

Theorem 2. If conditions 1 and 2 above hold, then the posterior of $\underline{\phi}_1$, given $\underline{W} = (\underline{W}_1, \underline{W}_2)$ does not depend on \underline{W}_2 , given that \underline{W}_1 and \underline{W}_2 are independent.

Proof. We have, since \underline{W}_1 and \underline{W}_2 are independent, and that conditions 1 and 2 hold, that

$$p(\underline{\phi}_1 | \underline{W}_1, \underline{W}_2) \propto \int_{\underline{\phi}_2} \int_{\underline{\theta}} a(\underline{\theta}) b(\underline{\phi}_1, \underline{\phi}_2) f_{\underline{W}}(\underline{w} | \underline{\phi}_1, \underline{\phi}_2, \underline{\theta}) d\underline{\theta} d\underline{\phi}_2 \quad (3.3a)$$

$$\propto \int_{\underline{\phi}_2} b(\underline{\phi}_1, \underline{\phi}_2) f_{\underline{W}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2) \left(\int_{\underline{\theta}} a(\underline{\theta}) f_{\underline{W}_2}(\underline{w}_2 | \underline{\theta}) d\underline{\theta} \right) d\underline{\phi}_2 \quad (3.3b)$$

$$\propto \int_{\underline{\phi}_2} b(\underline{\phi}_1, \underline{\phi}_2) f_{\underline{W}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2) d\underline{\phi}_2 \quad (3.3c)$$

where the inner integral in (3.3b), a function of \underline{w}_2 only, has been absorbed in the constant of proportionality, and the theorem is proved.

Note that the posterior of $\underline{\phi}_1$, given \underline{w}_1 alone, need not exist, even though the posterior of $\underline{\phi}_1$, given \underline{w}_1 and \underline{w}_2 is independent of \underline{w}_2 . For a direct computation of the posterior of $\underline{\phi}_1$, given (only) \underline{w}_1 , yields (under the assumptions of Theorem 2),

$$p(\underline{\phi}_1 | \underline{w}_1) \propto \int_{\underline{\phi}_2} \int_{\underline{\theta}} a(\underline{\theta}) b(\underline{\phi}_1, \underline{\phi}_2) f_{\underline{w}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2) d\underline{\theta} d\underline{\phi}_2 \quad (3.4a)$$

$$\propto \int_{\underline{\phi}_2} b(\underline{\phi}_1, \underline{\phi}_2) f_{\underline{w}_1}(\underline{w}_1 | \underline{\phi}_1, \underline{\phi}_2) \left\{ \int_{\underline{\theta}} a(\underline{\theta}) d\underline{\theta} \right\} d\underline{\phi}_2; \quad (3.4b)$$

but if the prior density of $\underline{\theta}$, $a(\underline{\theta})$ is improper, then $p(\underline{\phi}_1 | \underline{w}_1)$ does not exist. For further remarks on improper priors, see Dawid, Stone, and Zidek (1973).

An example is provided by the following:

$$\underline{w}_1 = \{(z_i, x_{2i}); i = 1, \dots, n\}, \text{ where the } z_i \text{ are constants,} \quad (3.5)$$

$$f(x_{2i} | z_i; \alpha, \beta, \tau^2) = (2\pi\tau^2)^{-1/2} \exp - \frac{1}{2\tau^2} (x_{2i} - \alpha - \beta z_i)^2,$$

and

$$\underline{w}_2 = \{x_{1j}^*, j=1, \dots, n^*\} \quad (3.6)$$

$$f(x_{1j}^* | \mu, \sigma^2) = (2\pi\sigma^2)^{-1/2} \exp - \frac{1}{2\sigma^2} (x_{1j}^* - \mu)^2,$$

where

$$\underline{\phi}_1 = (\alpha, \beta)'; \quad \underline{\phi}_2 = \phi_2 = \tau^2; \quad \underline{\theta} = (\mu, \sigma^2)' \quad (3.7)$$

Finally, we assume that

$$p(\underline{\phi}_1, \phi_2, \underline{\theta}) = a(\underline{\theta}) b(\underline{\phi}_1, \phi_2), \quad (3.8)$$

with

$$a(\underline{\theta}) = a(\mu, \sigma^2) \propto 1/\sigma^2 \quad (3.8a)$$

and

$$\begin{aligned} b(\underline{\phi}_1, \phi_2) &= b(d, \beta, \tau^2) \\ &= \frac{|A|^{1/2} S_0^{(n_0-2)/2}}{2^{n_0/2} (\Gamma(\frac{1}{2}))^2 \Gamma(\frac{n_0-2}{2})} (\tau^2)^{-(\frac{1}{2}n_0+1)} \exp \left\{ -\frac{1}{2\tau^2} [S_0 + Q] \right\} \end{aligned} \quad (3.8b)$$

where

$$Q = (\alpha - \alpha_0, \beta - \beta_0)' A (\alpha - \alpha_0, \beta - \beta_0) \quad (3.8c)$$

with A positive definite. It turns out that the posterior of (α, β) , given W_1 and W_2 , is connected to the distribution of a bivariate t , degrees of

freedom ($n+n_0-2$), which is independent of the x_{ij}^* 's, that is, of W_2 , and, further, that the posterior of ϕ_1 , given W_1 alone, does not exist - these results may be seen by substituting in (3.3c) and (3.4b).

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ABSTRACT (continued)

W_{sub 2} prime

distribution of ϕ does not depend on the data W_2 , which can thus be ignored. This also has consequences for the predictive distribution of future (or missing) observations. In fact, under similar conditions, the predictive distributions using W or just W_1 are identical.

W_{sub 1}

